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A program to create permeability fields that honor single-phase flow rate and pressure data

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Abstract

Accurate prediction (or simulation) of reservoir performance or contaminant transport in groundwater requires a realistic geological model representative of the reservoir/aquifer heterogeneity. Geostatistics provides tools for constructing such complex geological models constrained by different types of available (hard and soft) data and providing an assessment of related uncertainty. Permeability and flow data are nonlinearly related through the flow equations. Derivation of permeability models that honor flow response data is typically an inverse problem. This paper presents a FORTRAN program for generating permeability fields conditional to multiple-well single-phase flow rate and pressure data through an iterative inverse technique, called the sequential self-calibration (SSC) method. The SSC method is geostatistically-based, that is, it generates multiple equiprobable realizations that honor the input geostatistics of permeability and match pressure data for the given flow rate, under the given boundary conditions. The unique aspects of SSC are: (1) the master point concept that reduces the amount of computation, (2) a propagation mechanism based on kriging that accounts for spatial correlations of perturbations and (3) a fast method for computing all sensitivity coefficients within a single flow simulation run. Results from running the SSC code using an example data set are presented.

Code available at http://www.iamg.org/cGEditor/index.htm

Keywords: Inverse problem; Geostatistics; Heterogeneity; Flow simulation; Reservoir/aquifer modeling; Optimization; Sensitivity coefficients

1. Introduction

A realistic geological model representative of natural reservoir/aquifer heterogeneity is critical for accurate prediction of reservoir performance or contaminant transport in these reservoirs/aquifers. Geostatistical techniques are widely used for creating heterogeneous reservoir/aquifer models for such purpose (Journel, 1989; Deutsch and Journel, 1997). A geostatistical reservoir/aquifer model should incorporate as much available information as possible so that the predictions are more site specific with less uncertainty. In practice, the relevant information on reservoir heterogeneity may include static (hard or soft) data (such as conceptual geological data, well, log and core data or seismic data) and dynamic data (such as well test data, historical pressure data, fractional flow rate or saturation).

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Honoring static data is usually done through co-simulation techniques and a large variety of geostatistical techniques are available for such task (e.g. Alabert, 1987; Xu et al., 1992; Gómez-Hernández and Journel, 1993; Zhu and Journel, 1993; Gómez-Hernández and Wen, 1994; Xu, 1995; Wen, 1996). Dynamic flow data also carry important information, since they are direct measures of reservoir responses, which are directly related to the recovery process being used (Deutsch and Hewett, 1996; Wen et al., 1997b). Integrating dynamic flow data will further improve the accuracy of predictions and reduce uncertainty. However, honoring dynamic data in geostatistical reservoir models remains a challenge and is a very active area of research (e.g. Deutsch, 1992; Sahuquillo et al., 1992; Datta-Gupta et al., 1995; Reynolds et al., 1995; He et al., 1996; Oliver, 1996; Oliver et al., 1996; Wen et al., 1996, 1997a; Landa, 1997; Roggero, 1997; Tjelmeland, 1997).

The main difficulty in honoring dynamic data is the global and nonlinear relation between flow data and reservoir/aquifer properties through the flow equations. Thus, matching dynamic data is an inverse problem in which the flow equations must be solved to establish the relationship between data and model parameters (Tarantola, 1987; Sun, 1994). The conventional geostatistical techniques are usually not suited to integrate directly such data.

A popular method for solving an inverse problem is to pose it as an optimization (minimization) problem in which an objective function measuring the mismatch between observed data and model responses is minimized. The optimization method searches for optimal model parameters that best match the data, subject to constraints imposed by the flow equations and parameter variation. A number of inverse techniques have been developed in the literature (see review report from Wen et al., 1997b). Some of the limitations associated with many reported inverse techniques are:

- They are usually computationally intensive and limited to relatively small and simple models. They remain infeasible for practical application of realistic size problems.
- It is difficult to add in geostatistical constraints, i.e. they are not geostatistically based.
- No assessment of uncertainty.
- Often assume multi-Gaussian distribution of parameters or linear relationships between data and model parameters.

The sequential self-calibration (SSC) method is an iterative geostatistically-based inverse technique that allows generation of multiple equiprobable realizations of reservoir property models that match dynamic data, in addition to typical geostatistical constraints. This paper describes the FORTRAN computer code for SSC-generation of permeability models. The program presented here is limited to single-phase flow data that would be observed in many groundwater settings and in primary depletion or before water breakthrough in petroleum settings. The data we consider are flow rates $Q_w(t)$ and pressures $P_w(t)$ as a function of time, t, at any arbitrary number of well locations, $w = 1, \ldots, n_w$. Also the current SSC code presented is limited to the generation of a 2-D permeability model.

2. The SSC algorithm

The SSC algorithm is briefly described in this section. For more details, readers are referred to (Sahuquillo et al., 1992; Gómez-Hernández et al., 1997; Wen et al., 1997a). The required input information includes:

- geostatistical parameters of permeability field including histogram, variogram and other related statistics,
- hard and soft static permeability data, if any,
- initial and boundary conditions for flow,
- time-dependent flow rate $Q_w(t)$ and the observed pressure $P_w(t)$ data at well w at time t.

The goal is to create a permeability model at the given scale that honor the given geostatistical parameters and static data, and match the pressure data under the given flow rate and boundary conditions. In

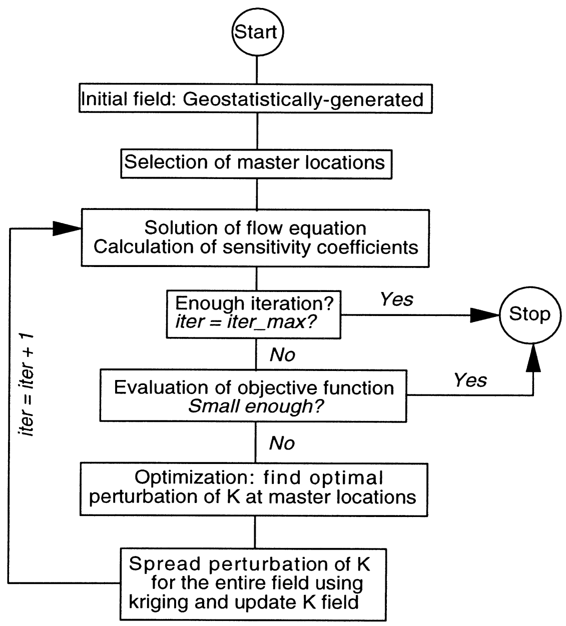

Fig. 1. Flowchart of sequential self-calibration method.

summary, the main steps in the SSC method include (see Fig. 1):

- Construct multiple initial permeability realizations that honor as much static (hard and soft) information as possible. The statistics (e.g. histogram and variogram) at the required scale are needed. A Gaussian technique such as sequential Gaussian simulation (see the `sgsim` program in GSLIB, (Deutsch and Journel, 1997) has been used in the examples presented later. Each initial realization is processed one-at-a-time with the following steps.
- Solve the flow equations using the input well flow rates $Q_w(t)$, initial and boundary conditions. The bottom-hole pressure $\hat{P}_w(t)$ predicted from the simulator is compared to the known well pressures $P_w(t)$. An objective function is written that measures the squared difference between predicted and observed pressures:

  $$O = \sum_w \sum_t W_w(t)[\hat{P}_w(t) - P_w(t)]^2,$$

  with $W_w(t)$ being the weight assigned to different pressure data according to their accuracy. Matching of flow data is achieved by minimization of this objective function. A gradient method is used, which requires calculation of 'sensitivity coefficients,' that is, the derivatives of pressure with respect to the permeability perturbation:

  $$\frac{\partial P_w(t)}{\partial \Delta K_i} \quad i = 1, \ldots, N,$$

  where N is the number of blocks in the model. In practice, the number N of actual block permeabilities being perturbated is reduced to between 1/10 and 1/100 of the number of blocks, by using the 'master point' concept (RamaRao et al., 1995; Gómez-Hernández et al., 1997). Note that the actual number of master points depends on the correlation range of permeability. Optimal changes of permeability are determined at these 'master points' and then smoothly interpolated by kriging to all grid blocks.The sensitivity coefficients are calculated as part of the solution of the flow equations. The detailed description of the method for computing the sensitivity coefficients is given in Wen et al. (1997a).
- Determine optimal perturbations of permeability values at all master locations using a modified gradient projection method (Gómez-Hernández et al., 1997). This 'inner optimization' determines the permeability changes that would lead to more closely matching of the pressure data at all wells at all times. The mathematical description of this optimization procedure can be found in Gómez-Hernández et al. (1997).
- The optimal permeability perturbations at the master point locations are smoothly propagated to all grid cells by kriging.
- Iterate until the objective function is sufficiently close to zero, or the maximum number of outer iterations has been exceeded. Fewer than 20 iterations are normally required.

The unique features of the SSC algorithm are: (1) the concept of master point that reduces the parameter space to be estimated, (2) the propagation of perturbations through kriging and (3) the fast computation of sensitivity coefficients of pressure within one single phase flow simulation run that makes inversion feasible.

This inversion procedure results in distributions of permeability that are consistent with the flow data and spatial geostatistical characteristics of the initial permeability models.

3. Implementation details

The `ssc` program described hereafter tries to balance the two competing goals of (1) being robust and easy to use and (2) being flexible with respect to input data and boundary conditions. This source code provides an advanced starting point for further development. Many of the assumptions stated below can be relaxed.

Consider a rectangular 2-D domain regularly gridded with square blocks. The effect of gravity is assumed constant, that is, the domain is implicitly assumed to be a horizontal plane. Porosity is assumed to be constant; only variations in permeabilty are considered. The boundary conditions at the four boundaries are either constant flow rate, constant pressure, or a combination of the two. The flow equations solved by `ssc` are for single-phase, unsteady-state, slightly compressible flow.

The `ssc` code uses natural logarithm of permeability, ln(K). Thus the geostatistics of ln(K) (e.g. histogram and variogram) are required for input. Permeability is assumed hydraulically isotropic (i.e. permeability at each cell is a scalar). The master point locations are randomly selected and their locations changed after each 3–4 (chosen by users) outer iterations. The number of master points required depends mainly on the number of wells and the correlation length of the permeability field: more master points are required for more wells or short correlation length. Our experience shows that two to three master points per correlation range in each direction are sufficient for most applications (Capilla et al., 1997, 1998).

Conditioning to static data is performed by including the conditioning data points as master points with their constraint intervals in the optimization being related to the softness of the data (Wen, 1996; Capilla et al., 1997).

The flow simulation is performed by imposing the boundary conditions and the input single-phase flow rates at the injection and production wells; thus, the entire rate history of each well must be specified. A block-centered 5-point finite-difference method is used to solve the flow equations. A direct band-matrix solver is utilized to solve the system of equations for pressure values at the center of each block. The pressure at the well is determined from the block pressure using Peaceman's formula (Peaceman, 1977). The objective is to match the measured pressures at the injection/production wells. Flow simulation provides a completed record of pressure versus time at all locations in the reservoir; however, the objective function only considers the pressures at prescribed locations and times. There is no requirement for a full pressure history at each well.

The program iteratively searches for the optimal perturbations of $\ln(K)$ at master locations and modifies the entire input realizations until the pressures are matched within a specified tolerance or a maximum number of iterations has been exceeded. A linear relation between pressure and permeability perturbation is assumed when searching for these optimal perturbations based on the sensitivity coefficients.

The spreading of perturbations at the master points to the entire domain is based on kriging. Simple kriging (SK) or ordinary kriging (OK) can be used for this purpose depending on the number of hard data: if the number of hard data is small, simple kriging is preferred, otherwise ordinary kriging is suggested.

The original (input) realizations can be generated by geostatistical programs such as `sgsim` in GSLIB (Deutsch and Journel, 1997) or other codes. The more closely these input realizations match the true spatial features, the faster the `ssc` program will converge to a solution that matches the pressure data. The histogram of $\ln(K)$ can be reproduced explicitly by performing a transformation after each modification of the permeability field (Xu, 1995).

Adding a smooth correlated perturbation field to the initial permeability field does not guarantee the reproduction of the variogram model in the updated field. A posterior check of the variogram is usually required to ensure that the variogram computed from the updated realizations is close to the initial realizations. A better way that can explicitly account for the variogram model is to propagate the optimal perturbations at the master points through conditional simulation, which is easy to apply, but not included in this version. The `ssc` program is written in FORTRAN 77 standard. The ANSI standard has been adhered to as closely as possible to ensure smooth compilation on a variety of platforms. Program input and output are through ASCII files. Some other minor implementation issues, such as the selection of the constraint interval at master points, optimization procedure, and accounting for uncertainty in measured data, are discussed to related papers (Wen, 1996; Capilla et al., 1997, 1998).

4. Program parameters

This `ssc` program is a research code suitable for experimenting with the ideas presented in the related papers; the program has not been coded for ultimate speed and it has not been thoroughly debugged. The `ssc` program loosely follows GSLIB conventions. The parameters required for the program are listed next and shown in Fig. 2:

- Line 1: input file containing local well conditioning data (X-coordinate, Y-coordinate and natural logarithm of permeability, $\ln(K)$). The standard GSLIB/GeoEAS format is expected (Deutsch and Journel, 1997).
- Line 2: columns in the well data file for the X- and Y-coordinates, permeability values, and the associated measurement error. The coordinates should match the grid definition (see later) and permeability should be in units of milli-Darcies (md). The measurement error is in terms of variance.
- Line 3: number of permeability data and number of wells with flow data.
- Line 4: index for matching a target histogram given by the next input file.
- Line 5: input file containing the input/target $\ln(K)$ histogram.
- Line 6: columns in the $\ln(K)$ histogram file for permeability and possibly declustering weight.
- Line 7: mean (md) and variance (md^2) of $\ln(K)$ field. These are used if no input permeability distribution is specified; they are then taken as parameters for a lognormal distribution of permeability.
- Line 8: input file with reservoir and well data. The first line in this file contains porosity, thickness (feet), viscosity (cp), and compressibility (1/cp). The following `nwell` lines are the (i, j) location of the well and its radius (feet).
- Line 9: input file with time (days) and the measured flow rate (STB/day) data at all wells (negative for production, positive for injection). This file may contain missing values. All time steps with measured pressure data should be included. The flow rate is assumed constant between two time-step values; a new value changes the rate.

- Line 10: input file with time, the measured pressure data (psi), and the associated weight at all wells.
- Line 11: input file with index for boundary conditions and the corresponding boundary values (pressure, psi or flow rate, STB/days): 1 for constant pressure boundary and 0 for constant flow rate boundary. The type of boundary for upper, left, right, and bottom boundaries are specified first, followed by their pressure or/and flow rate values.
- Line 12: input file with initial pressure field before production/injection. This input file is in GSLIB grid format (one value per line — X cycling fastest, then Y).
- Line 13: input file with initial ln(K) realizations (obtained from `sgsim` or some other geostatistical algorithms). Constant permeability initial realizations could be used; but, some aspects of uncertainty will be lost and more iterations may be required for convergence. The input file is in GSLIB grid format (one value per line — X cycling fastest, then Y, then by realization). The program expects the permeabilities to be in the first column.

```
              Permeability Inversion with Sequential Self Calibration
              *******************************************************

START OF PARAMETERS:
wells.dat           -file with local well conditioning ln(K) data        - line  1
1  2  3  4          -columns for X, Y coordinate, Perm. & Error          - line  2
0  3                -num. of ln(K) data and num. of wells with flow data - line  3
1                   -index for identifying desired histogram             - line  4
ref_ave.dat         -file with ln(K) histogram (scale of SSC model)      - line  5
1  0                -columns for permeability and weight                 - line  6
3.0     2.0         -mean and variance of ln(K) distribution             - line  7
wellpara.dat        -file with reservoir and well data                   - line  8
flowrate.dat        -file with input flow rate and time step data        - line  9
pressure.dat        -file with input pressure data                       - line 10
boundary.dat        -file with boundary conditions                       - line 11
pinit.dat           -file with initial pressure for the entire field     - line 12
kinit.dat           -file with input realizations                        - line 13
100 1   100         -number of realizations                              - line 14
-999.   1.0e21      -trimming limits for missing values                  - line 15
1                   -debuging level                                      - line 16
ssc.dbg             -file for debug output                               - line 17
ssc.out             -file for output ln(K) permeability realizations     - line 18
obj.out             -file for output objective function after each iter  - line 19
prematch.out        -file for output matching of pressure responses      - line 20
20 100.0   200.0    -X grid size: nx, xmn, xsiz                          - line 21
20 100.0   200.0    -Y grid size: ny, ymn, ysiz                          - line 22
38774               -random number seed                                  - line 23
4    4              -number of master points in X and Y (random strat.)  - line 24
3                   -number of outer iterations to update master points  - line 25
3.0                 -factor for defining constraint interval for opti.   - line 26
10  0.3 0.001       -max num of outer iter, dampening para  & min  tol   - line 27
50 5e-4 5e-3 40     -optimization parameters (see text)                  - line 28
2000                -search radius                                       - line 29
1     24            -ndmin,ndmax                                         - line 30
0                   -type of kriging (0: SK, 1: OK)                      - line 31
1 0.0               -variogram Model: num struc, nugget effect           - line 32
1 1.0 90. 1800 400  -type, sill, azm, max range, min range               - line 33
```

Fig. 2. Parameter file for `ssc` program.

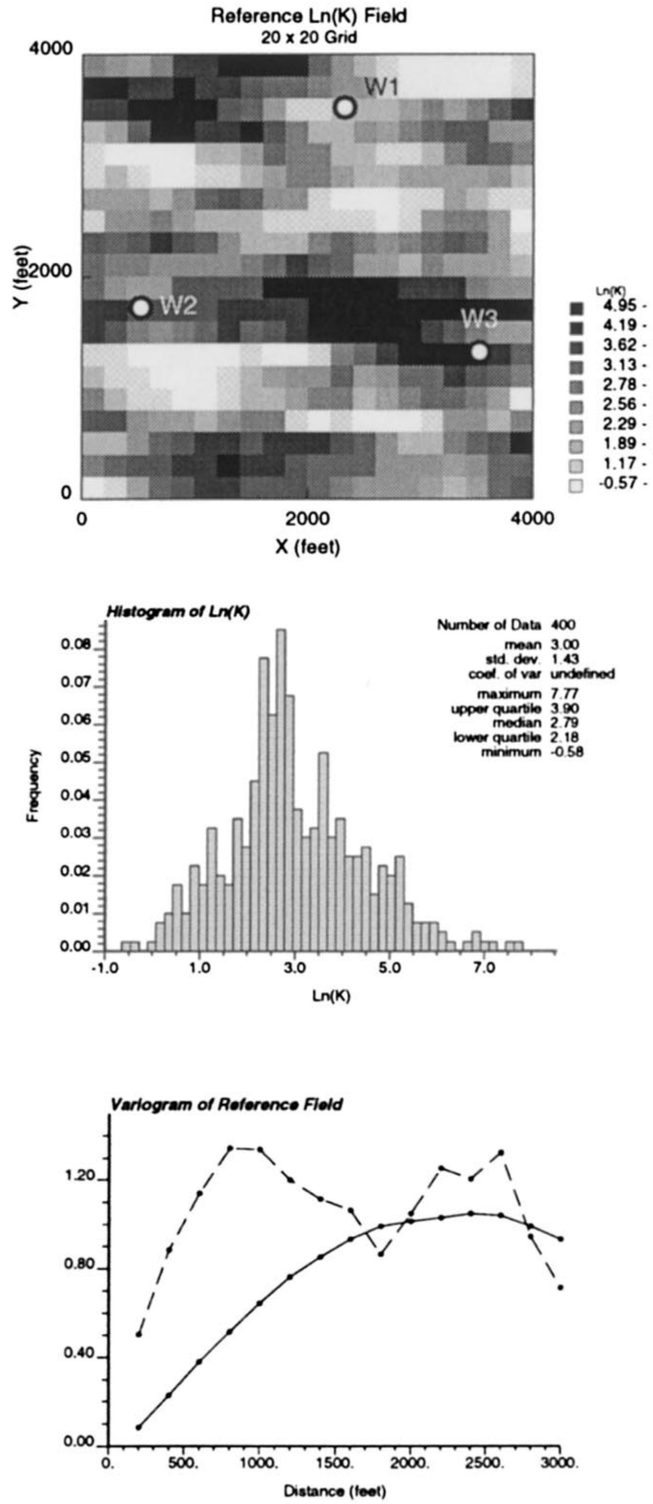

Fig. 3. Reference ln(K) field and corresponding histogram and variogram (solid line for X-direction, dash line for Y-direction).

- Line 14: total number of realizations in the file of initial $\ln(K)$ field, the first and last realization numbers used for updating.
- Line 15: trimming limits used to flag missing values; permeability, rate, or pressure data with a value less than the lower limit or greater than the upper limit are discarded.
- Line 16: index for debugging level. A high number indicates more information output in the debugging file for more serious debugging.
- Line 17: output file for debugging messages including the input data and their reproduction at each iteration, the master point locations for each iteration, the sensitivity coefficients and the optimal ΔK values at each iteration.
- Line 18: output file for final updated $\ln(K)$ permeability realization(s). The realizations are written from the lower left corner and then realization-by-realization (X cycles fastest, then Y and then realization number).

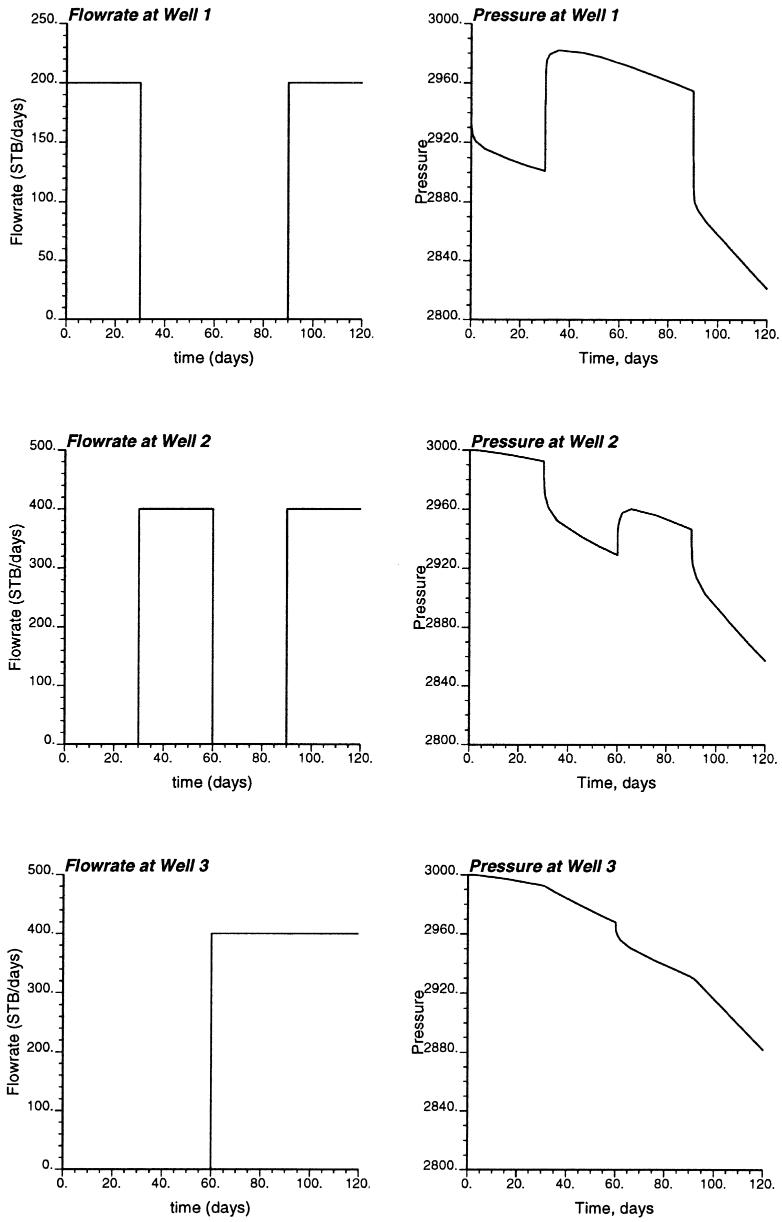

Fig. 4. Flow rate at three wells and corresponding pressure responses computed from reference field.

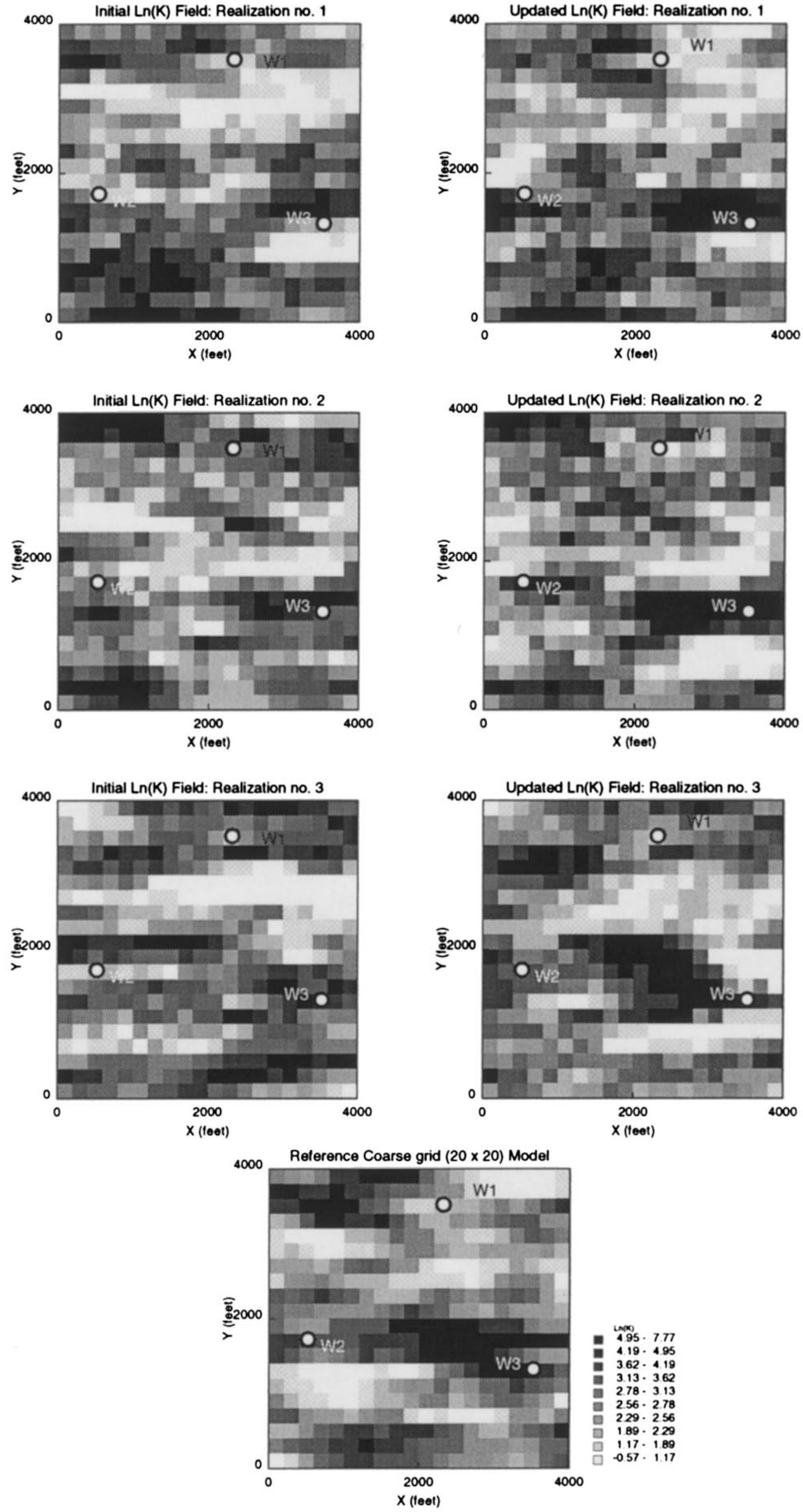

Fig. 5—*Caption opposite*

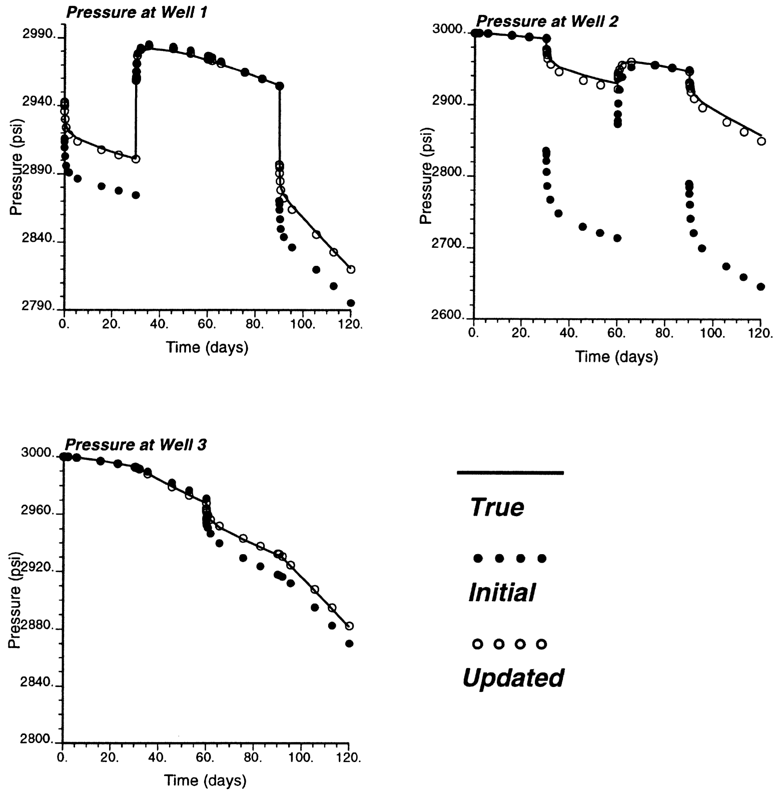

Fig. 6. Pressure responses computed from initial and updated permeability fields compared to reference data.

- Line 19: output file for changes of the objective function value with iteration number and the final deviations of simulated and observed pressure data.
- Line 20: output file for comparison of observed pressure data and simulated pressures from initial and updated ln(K) fields.
- Line 21: the size of the model in the X-direction (number of blocks, center of first block and block size in feet).
- Line 22: the size of the model in the Y-direction (number of blocks, center of first block, and block size in feet).
- Line 23: the random number seed (a large integer) for generating random master point locations.
- Line 24: the number of master points in the X- and Y-directions. The 2-D study area is divided into a regular grid and a random stratified sampling scheme is considered to arrive at the master point locations. Note that the well locations are also considered as master point locations.
- Line 25: the number of outer iterations after which the master point locations are randomly reselected.
- Line 26: a constant factor used for defining the constraint interval of permeability at master locations in the optimization.
- Line 27: the maximum number of outer iterations, the relaxing (dampening) parameter for each outer iteration and the minimum (target) objective function for convergency control (the initial objective function is normalized to start at 1.0).
- Line 28: parameters used for determining the optimal changes of permeability at the master point locations: (1) minimum iteration number, (2) tolerance for norm 1 of pressure, (3) the minimum difference of objective function in two consecutive iterations, (4) the maximum number of times that the difference of objective function in two consecutive iterations is smaller than the value specified as previous parameter.
- Line 29–33: kriging parameters and variogram

Fig. 5. Three initial permeability realizations and corresponding updated fields from SSC inversion. True reference field is shown at bottom.

model to propagate the changes of permeability: (1) radius of search neighborhood, (2) minimum and maximum number of data for kriging system, (3) type of kriging (0 is simple kriging, 1 ordinary kriging), (4) number of nested structures excluding the nugget effect, (5) the nugget effect, then, for each nested structure: (6) the type of structure (1 is spherical, 2 exponential, 3 Gaussian, 4 power law), (7) the direction of greatest continuity measured in degrees clockwise from north, (8) the range in the direction of greatest continuity and (9) the range in the direction perpendicular to the direction of greatest continuity.

5. An example

A simple application of SSC is now presented. A synthetic reference field is first set up with 20×20 grid. The size of each cell is 200×200 feet resulting in the entire field size being 4000×4000 feet. The reference permeability field ($\ln(K)$) is created by the sequential gaussian simulation (`sgsim`), see Fig. 3. The input mean and variance are 3.0 and 2.0, respectively. The input variogram model is anisotropic spherical with ranges 1800 and 400 feet along X- and Y-directions. The histogram and variogram of that reference field are also given in Fig. 3. Three producing wells with production rates and the corresponding pressure responses shown in Fig. 4. The wells are turned on and off at different times on purpose to create as much between-wells interference as possible. Other reservoir parameters are: porosity $\phi = 0.2$, viscosity $\mu = 0.3$ cp, compressibility $c = 10^{-5}$ 1/psi, reservoir thickness $h = 100$ feet, initial pressure is constant with $p_0 = 3000$ psi and well radius is $r_w = 0.3$ feet. No-flow is assumed for all boundaries, which is frequently used in petroleum applications. The main features of the reference field are (1) the major spatial correlation is along the X-direction, (2) the high permeabilities of wells 2 and 3, while well 1 is located in a low permeability area.

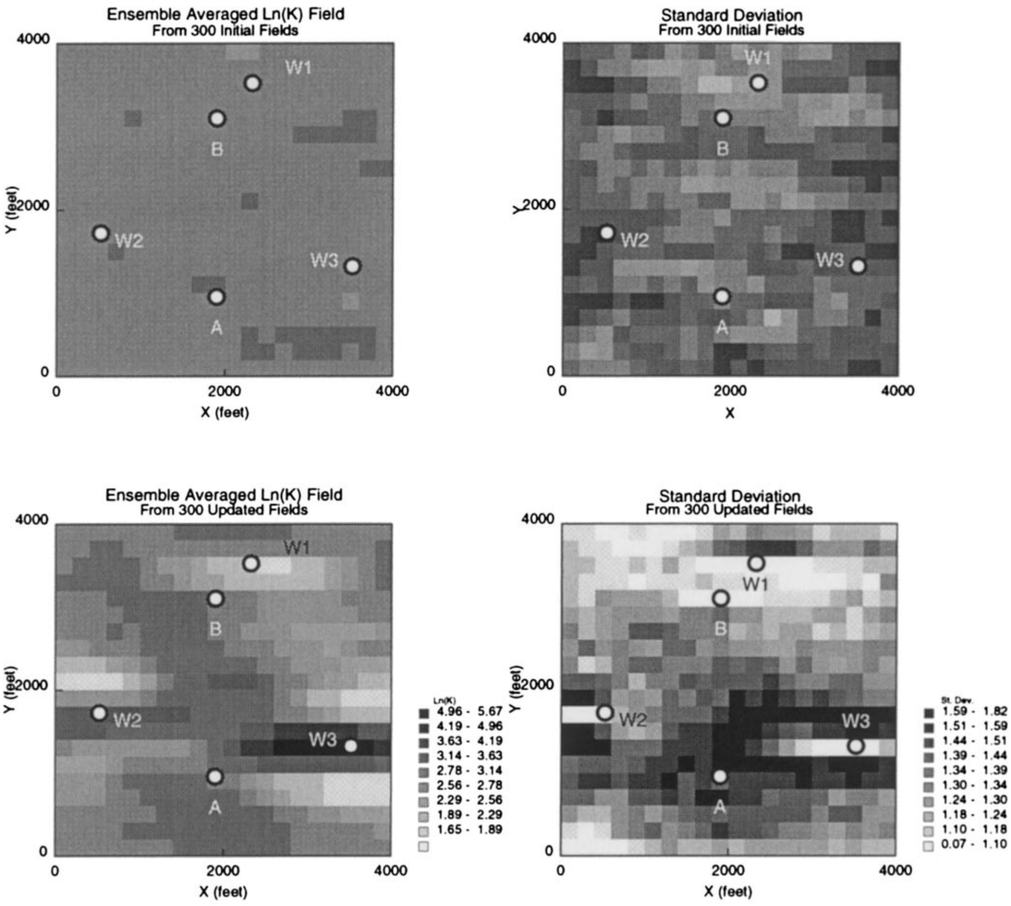

Fig. 7. Ensemble averaged permeability field and corresponding standard deviations from 300 initial and updated realizations.

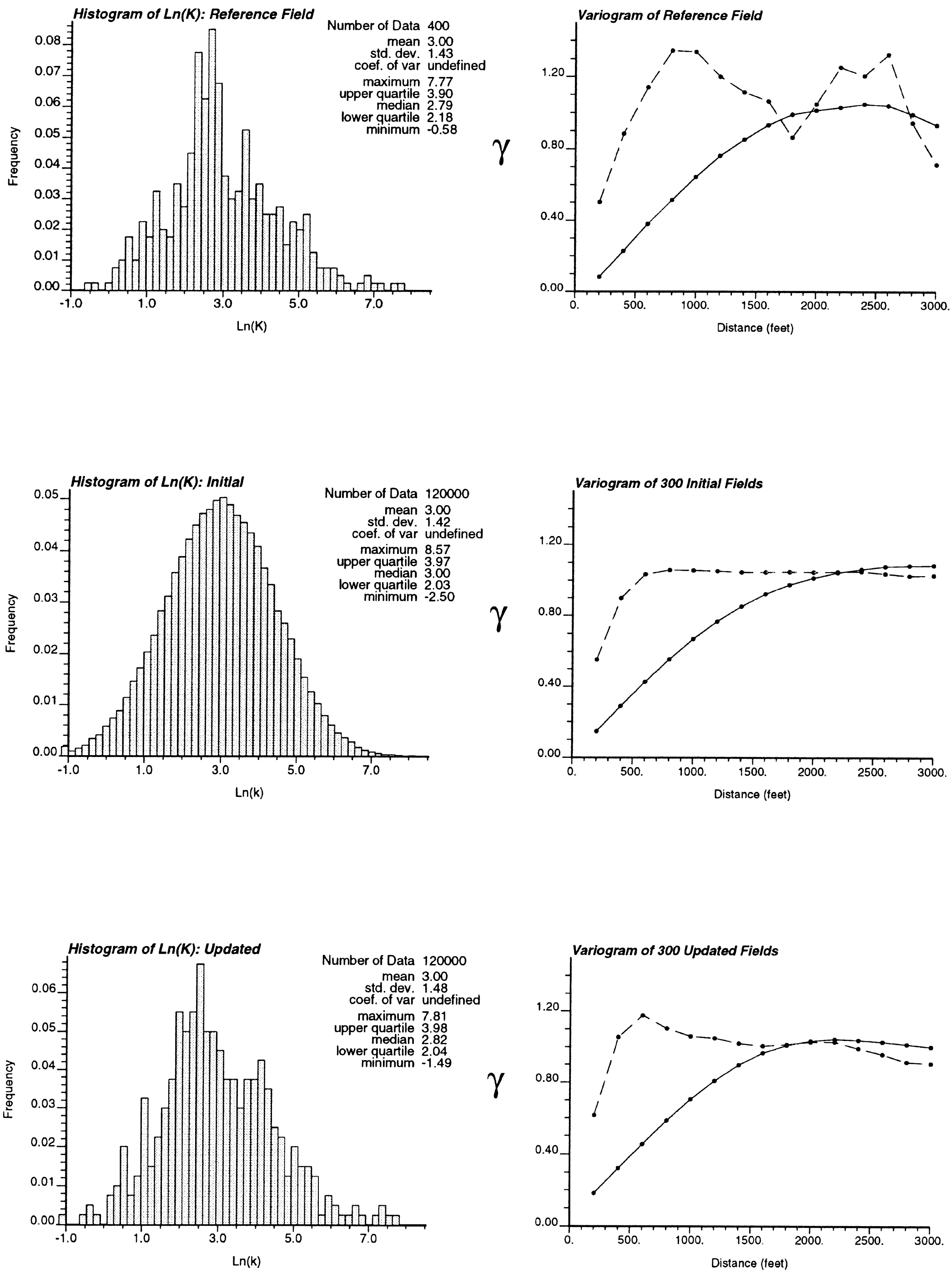

Fig. 8. Histograms and variograms of true permeability, 300 initial and updated permeability realizations. In variogram plots: solid lines for X-direction, dash lines for Y-direction.

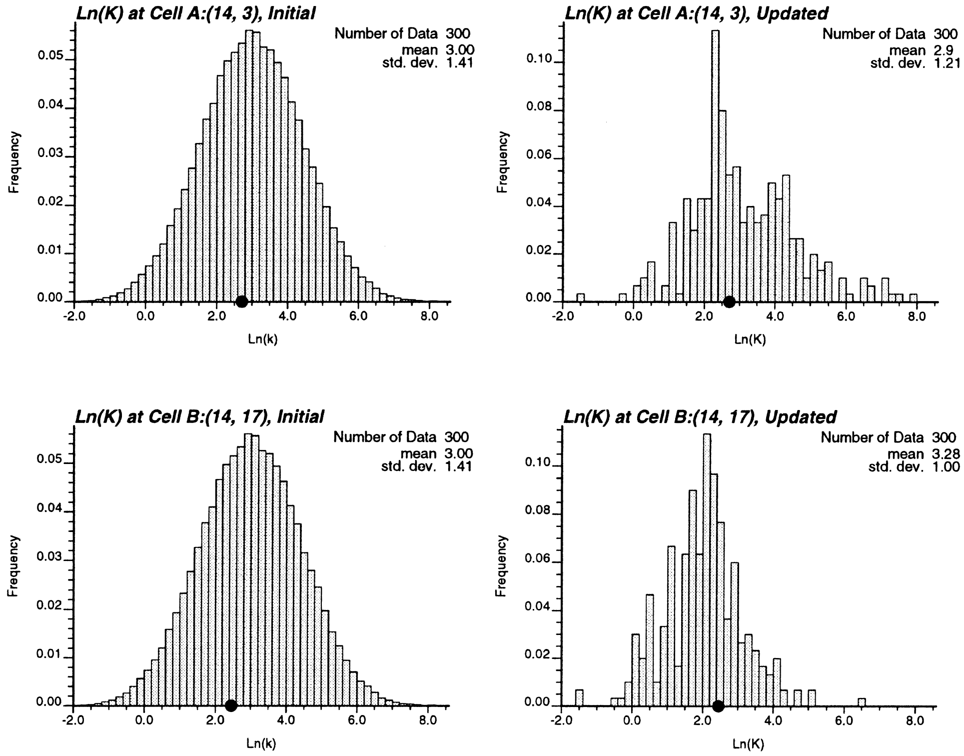

Fig. 9. Histograms of permeability values at locations A and B (see Fig. 7) computed from 300 initial and updated realizations. Bullets are true values from reference field at same locations.

Based on the reference histogram and variogram, we generate initial, unconditional, permeability realizations using `sgsim` assuming no permeability data is available; these realizations are then updated to match the flow data. Three initial and updated realizations are shown in Fig. 5. Thirty-two (4×8) randomly selected master points are used and their locations are updated every three outer iterations. The reference variogram model and histogram are used for the SSC inversion. The required CPU time for generating one realization is 3 min using a SGI Indigo workstation. The spatial features of the updated fields are closer to the reference field when compared to the initial fields, particularly (1) the permeability values around well 1 are always low, while permeabilities around wells 2 and 3 are updated to higher values, (2) wells 2 and 3 are spatially connected by high permeability values.

The matching of pressure data at the three wells for the initial and the updated permeability field (first realization) is shown in Fig. 6; the flow responses in the initial field are significantly different from the true responses, while the permeability field updated by SSC matches accurately the true responses.

As mentioned previously, SSC method is geostatistically-based, which allows uncertainty to be assessed by generating multiple equally likely permeability realizations. All realizations share the same histogram and variogram, and match flow data. The ensemble results of 300 initial and updated permeability realizations are shown in Fig. 7. Integrating flow data provides constraints on permeability distribution especially for large scale trend, allowing reduced uncertainty around well locations.

The histogram and variogram computed from the 300 initial and updated permeability fields are given in Fig. 8, which indicates the preservation of the geostatistic characteristics in the updated permeability realizations. The histograms of permeability values at two selected locations (A and B, see Fig. 7) from the 300 initial and updated realizations are given in Fig. 9, which indicates more accurate estimation with reduced uncertainty by integrating flow data, particularly, at near well area (e.g. location B in Fig. 9).

6. Conclusions

It is essential that reservoir/aquifer models of permeability honor dynamic flow data. This is the first public domain code for the SSC algorithm that allows permeability fields to be iteratively adjusted to honor flow rate and pressure data. The ssc program can handle injection and production data for an arbitrary number of wells and can use a constant flow rate or constant pressure boundary conditions.

Key assumptions in this ssc implementation include (1) the domain of interest is rectangular, 2-D and flat, (2) there is only single-phase flow.

The research program ssc provides a useful starting point for reservoir engineers and groundwater modelers considering the integration of flow data in their earth models.

7. Code availability

The source code and the example parameter and data files of ssc used for this paper are available through anonymous FTP from eluard.stanford.edu, mundo.upv.es under /pub/ssc, or from ftp.iamg.org.

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